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Graphical Knowledge Display – Mind Mapping and Concept Mapping as Efficient Tools in Mathematics Education

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In didactical discussion there is a widespread consensus that mathematics should be experienced by students as a network of interrelated concepts and procedures rather than a collection of isolated rules and facts. This experience may be supported by representing mathematical knowledge graphically in the form of networks.

In this paper, two special graphical representations of mathematical networks, mind maps and concept maps, are presented. Both are means to show ideas and concepts connected with a topic. Their suitability as a pedagogical tool for mathematics education is considered and the possible applications of mind mapping and concept mapping in mathematics education together with their advantages and limits are discussed. It turns out that both, mind mapping and concept mapping, may be efficient tools to improve mathematics achievement.

1. Introduction

Mathematical knowledge has the character of a network, as mathematical objects, i.e. concepts, definitions, theorems, proofs, algorithms, rules, theories, are interrelated but also connected with components of the external world. Accordingly, there is a widespread consensus in the didactical discussion that mathematics should be experienced by students in its interrelatedness (see e.g. NCTM Yearbook 1995, Preface, or NCTM Principles and Standards for School Mathematics 2000, p.64). The importance of this notion also becomes apparent in the recent PISA–Study, where interconnections and common ideas are central elements (OECD, 1999, p.48).

One means to experience the network character of mathematics is by visualising it. Two methods especially suited for representing graphically a mathematical network around a topic are mind mapping and concept mapping. These two techniques are presented below; their suitability as a pedagogical tool for mathematics education is considered and the possible applications of mind mapping and concept mapping in mathematics education together with their advantages and limits are discussed.

2. Mind mapping

2.1 Background

Mind mapping was firstly developed by Tony Buzan, a mathematician, psychologist and brain researcher, as a special technique for taking notes as briefly as possible whilst being interesting to the eye as possible. Since then, mind mapping turned out to be usable in many different ways other than just simple note taking. Mind maps have, among other things, been used in education, but despite their usefulness (see 2.2) are surprisingly rarely used in mathematics.

The method of mind mapping takes into account that the two halves of the human brain are performing different tasks. While the left side is mainly responsible for logic, words, arithmetic, linearity, sequences, analysis, lists, the right side of the brain mainly performs tasks like multidimensionality, imagination, emotion, colour, rhythm, shapes, geometry, synthesis. Mind mapping uses both sides of the brain (Buzan, 1976), letting them work together and thus increases productivity and memory retention. This is accomplished by representing logical structures using an artistic spatial image that the individual creates. Thus mind mapping connects imagination with structure and pictures with logic (Svantesson, 1992, p. 44; Beyer, 1996).

2.2 Rules for making mind maps

Mind maps are hierarchically structured. They are produced following the rules given below (see e.g. Beyer, 1993; T. Buzan & B. Buzan, 1993; Hemmerich et al., 1994; Hugl, 1995, p. 182; Svantesson, 1992, p. 55-56):

- Use a large sheet of paper without lines in landscape format.
- Place the topic of the mind map in the centre of the paper. (The topic of the mind map should be displayed in an eye-catching way, preferably by a coloured image. If a picture does not seem appropriate, the topic should be named by a well-chosen keyword.)
- From the topic draw a main branch for each of the main ideas linked to the topic. Write keywords denoting the main ideas directly on the lines. Use printed letters. (The order of the branches is not important. If a special order is needed for understanding the topics, the branches may be

numbered or ordered clockwise. If possible, only one word per line, preferably a noun, should be written down. As 90% of the words in texts are unnecessary, using a few meaningful keywords will be sufficient to remember the entire context.)

- Starting from the main branches you may draw further lines (sub-branches) for secondary ideas (sub-topics) and so on. The order follows the principle: from the abstract to the concrete, from the general to the special.
- Use colours when drawing a mind map.
- Add images, sketches, symbols, such as little arrows, geometric figures, exclamation marks or question marks, as well as self-defined symbols to your mind map.

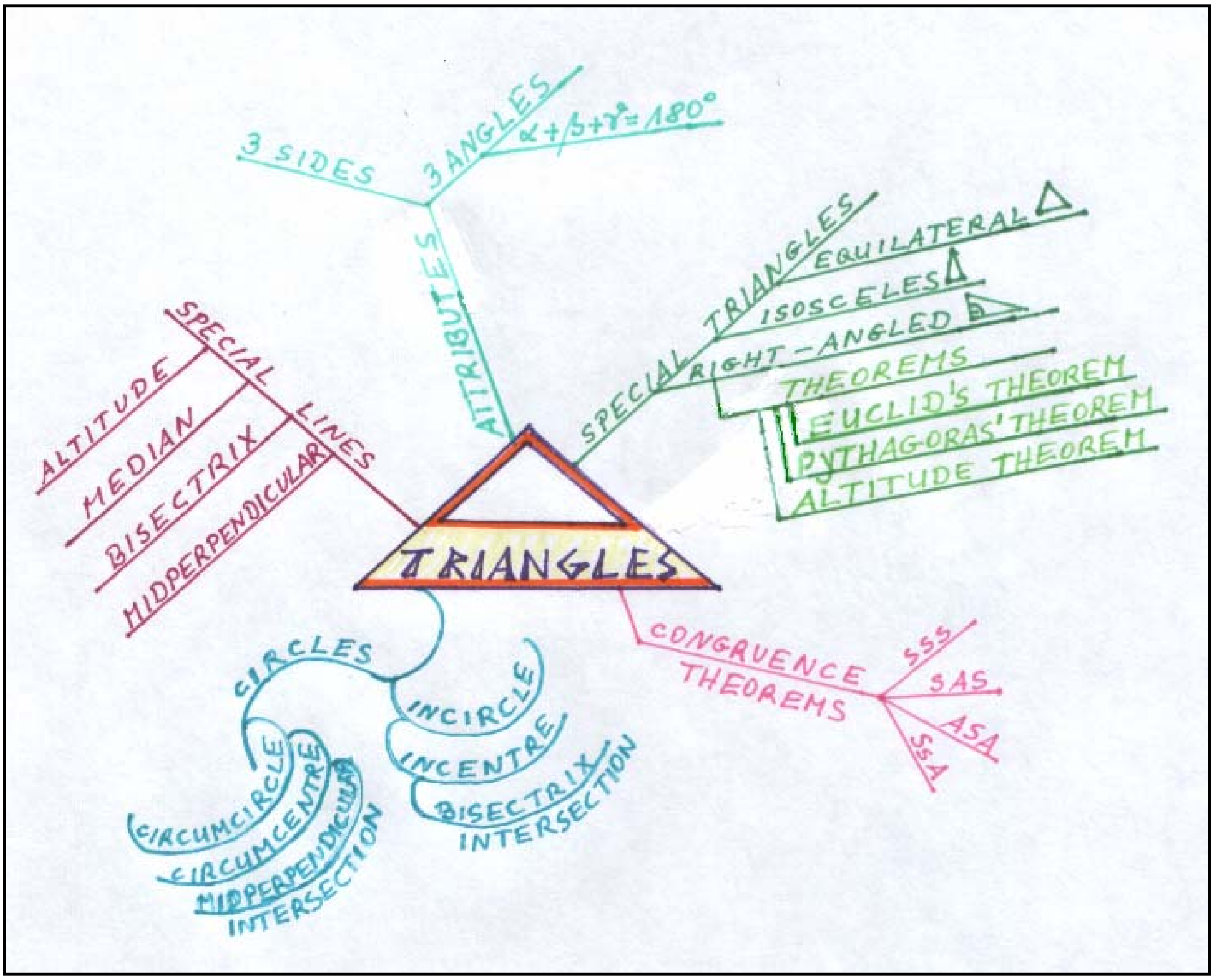

Figure 1: Mind map on the topic of triangles

2.3 Mathematical mind maps

Both, the structure of a mind map and the technique of mind mapping emphasise the usefulness of mathematical issues as topics for mind maps (Brinkmann, 2000, 2001b, 2002, in press).

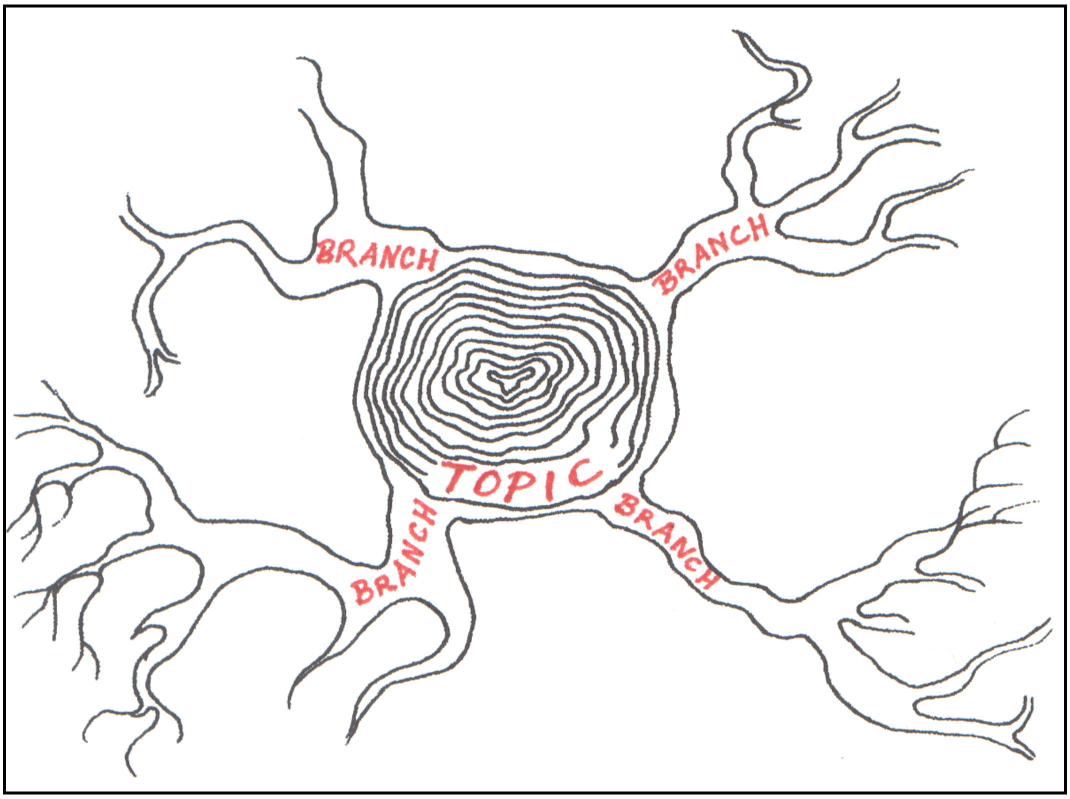

Figure 2: Structure of a mind map

The structure of a mind map resembles a tree seen from the top (figure 2): from the trunk in the middle, representing the topic of the mind map, the lines for the ideas linked to the topic branch off like tree branches. Thus a mind map is structured similarly to mathematics: "Mathematics is often depicted as a mighty tree with its roots, trunk, branches, and twigs labelled according to certain sub disciplines. It is a tree that grows in time" (Davis & Hersh, 1981, p. 18). Relations between mathematical objects may thus be visualised by mind maps in a structured way that corresponds to the structure in mathematics (Brinkmann 2000, 2001a).

The special technique of mind mapping, which uses both sides of the brain and has them working together, is of benefit to mathematical thinking, which takes place in both sides of the brain. The left hemisphere is better suited for analytic deduction and arithmetic, the right hemisphere for spatial tasks, e.g. geometry. The constant emphasis in mathematical education on rules and algorithms which are usually sequential may prevent the development of creativity and spatial ability (Pehkonen, 1997). Thus "the balance between logic and creativity is very important. If one places too much emphasis on logical deduction, creativity will be reduced. What one wins in logic will be lost in creativity and vice versa" (Pehkonen, 1997; see also Kirckhoff, 1992, p. 2). Accordingly, Davis and Hersh (1981, p. 316) "suggest that in mathematics it would be better for the contributions of the two halves of the brain to cooperate, complement, and enhance each other, rather than for them to conflict and interfere."

2.4 Uses of mind mapping in mathematics education

Some of the most important uses mind mapping may have in mathematics education, are listed below.

- ***Mind maps help to organise information.***

The hierarchical structure of a mind map conforms to the general assumption that the cognitive representation of knowledge is hierarchically structured (Tergan, 1986). Mathematical knowledge may thus be organised in a mind map according to this knowledge's mental representation. A clear and concise overview of the connectedness of mathematical objects around a topic is enabled. Moreover, this is supported by the use of colours and pictures.

In addition, mind mapping supports the natural thinking process, which goes on randomly and in a non-linear way. As mind maps have an open structure, one may just let one's thoughts flow; every produced idea may be integrated in the mind map by relating it to already recorded ideas, and this with virtually no mental effort.

- ***Mind maps can be used as a memory aid.***

Each mind map has a unique appearance and a strong visual appeal. Thus information may be memorised and recalled faster, the learning process is speeded up and information becomes long living.

- ***Mind maps can be of help to repetition and summary.***

At the end of a teaching unit the subject matter of the treated topic can be repeated and structured by composing a mind map; this mind map then serves as a good memorisable summary.

- ***A mind map may summarise the ideas of several students.***

A mind map may grow as the common task of an entire class: The teacher might write the topic in the middle of the chalkboard and ask the students what main ideas they connect with it. For each idea the teacher draws a main branch of the mind map. Further on, students are asked to tell all other ideas they link to these main ones. Due to the open structure of a mind map, each single contribution can be integrated. The complete mind map should be redrawn by each student in his or her own personal style.

- ***Mind maps help meaningfully connect new information with given knowledge.***

New information can be integrated into an existing mind map and related to previously learned concepts. Such an activity with students has to be initialised by the teacher, who has the overview of already created mind maps and of how new concepts fit to old topics.

- ***New concepts may be introduced by mind maps.***

Entrekin (1992) reports that she used mind maps to introduce new concepts in mathematics classes. The new concept "is written on the chalkboard or transparency. As the concept evolves in later lessons, the teacher may add additional components and form an extended mind map. This visual representation serves to help students relate unknown concepts to known concepts."

- ***Mind maps let cognitive structures of students become visible.***

Mind maps drawn by students provide information about the students' knowledge. In broad outline, a learner's knowledge structure gets visible by means of mind maps for both the teacher and the learner.

  - The student develops an awareness of his or her own knowledge organisation.

This process might be enhanced by having the students construct mind maps in small groups. The students the have to discuss the concepts to be used and the connections to be drawn.

  - Wrong connections in a students' knowledge become visible and can be corrected by the teacher. It is recommended to first ask the student why the (wrong) connection was drawn; the explanation given by the student might bring more insight into the underlying cognitive structure than the simple and reduced representation in the map.
  - The students' growth in the understanding of a topic can be checked when asking them to create both a pre- and a post-unit mind map (Hemmerich et al., 1994). The teacher might see e.g. if supplementary concepts are linked to the topic, in a meaningful way.

- ***Mind maps foster creativity.***

Everybody may develop a personal style for mind mapping. Mind maps may have different forms and shapes, different colours, symbols or images. Artistic arrangements are not only allowed but desired as advantageous. This leads to a gain in creativity and moreover gives great pleasure. The fostering of creativity has a positive effect on mathematical achievement. It is common experience that in schools where emphasis is placed on creative activities such as working on arts, music or literature, the students are also better in mathematics (Svantesson, 1992, p. 26).

- ***Mind maps may show the connections between mathematics and the "rest of the world".***

As a mind map is open to the addition of any idea someone associates with the main topic, non-mathematical concepts may also be connected with a mathematical object (see figure 3). Thus it becomes obvious that mathematics is not an isolated subject but is related to the most different areas of the "rest of the world" (Brinkmann, 1998, 2001c).

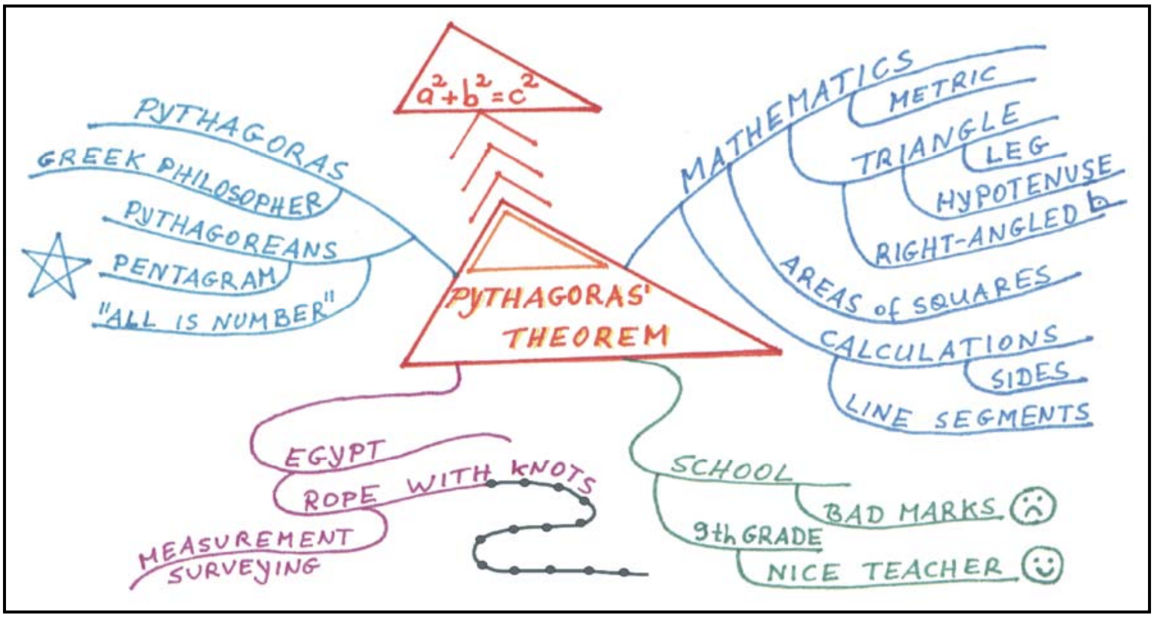

Figure 3: Mind map on the topic of the theorem of Pythagoras.

2.5 Limitations

In spite of its well-structured and ordered contents a mind map may sometimes, appear confusing. Mind maps are very individual graphic representations. As different people have different associations with the same topic they also draw different mind maps. The correct grasp of the relations represented in a mind map affords the right associations to the used key words. Hence, any mind map that someone wants to use should be drawn by that individual or group to which the individual belongs.

In a mind map, each main branch builds up a complex whole with its sub branches. Connections between the single aspects are not drawn in order to increase the clarity of the map. Thus, the relations in the mind map are probably incomplete.

3. Concept mapping

3.1 Background

Concept maps were first introduced by Novak as a research tool, showing in a special graphical way the concepts related to a given topic together with their interrelations. The method of concept mapping "has been developed specifically to tap into a learner's cognitive structure and to externalise … what the learner already knows" (Novak & Govin, 1984, p. 40), according to Ausubel's statement: "The most important single factor influencing learning is what the learner already knows. Ascertain this and teach him accordingly" (Ausubel et. al., 1980).

Although the primary intention was to use concept mapping in research, it was found this to be a useful tool in a variety of applications, including helping students to "learn how to learn" (Novak & Govin, 1984; Novak, 1990, 1996). Consequently, concept mapping has been used also as an educational tool, above all in science, especially in biology. The instruction and use of concept mapping in science is now well documented, but less comprehensively so in mathematics (Malone & Dekkers, 1984, p. 225; Hasemann & Mansfield, 1995, p. 47).

3.2 Rules for making concept maps

Concept maps are similar to mind maps. They are hierarchically structured, according to the assumption that the cognitive representation of knowledge is hierarchically structured (Tergan, 1986). A concept map is constructed according the following rules (see e.g. Novak & Govin, 1984).

- Use a large sheet of paper for your concept map.
- Position the topic at the head of the map.
- Arrange the other concepts beneath it on several levels, the more inclusive, general, abstract concepts at the top, the more specific, concrete concepts lower down. (It is helpful to transfer first these concepts to small pieces of paper and arrange these on the different hierarchy levels you see. There may be more than one valid way in ranking the concepts, depending on how you interpret the relationships between ideas.)
- If possible, arrange the concepts so that ideas go directly under ideas that they are related to. (Often this is not possible because ideas relate to several other concepts.)
- Note beneath the last row some examples to the concepts situated here.
- Draw lines from upper concepts to lower concepts that they are related to; do the same for any related concepts that are on the same level. (You may decide to rearrange the concepts during this stage; sometimes two or three

reconstructions are needed to show a good representation of the meaning as you understand it.)

- On the connecting lines, write words or phrases that explain the relationship of the concepts. (This is the most important and most difficult step! You may continue to rearrange the concepts to make the relationships easier to visualise.)
- Sometimes it is useful to apply arrows on linking lines to point out that the relationship expressed by the linking word(s) and concepts is primarily in one direction.
- Beneath the last row, put examples to the concepts situated here and connect the examples with the concepts they belong with. As linking words write a phrase like "for example".
- Copy the results of the above steps onto a single sheet of paper.
- Draw borders around the concepts. Do not draw borders around the examples.

3.3 Uses of concept mapping in mathematics education

Concept maps have been found to be useful in a variety of applications, in the teaching of the different sciences but also of mathematics at all levels ranging from primary school to senior high school. Concept maps can be used for example in the following situations (Novak & Govin, 1984; Novak, 1990, 1996; Malone & Dekkers, 1984):

- ***Concept maps help to organise information on a topic.***

In order to be useful, knowledge must be organised so as to facilitate understanding and problem-solving ability. A concept map organises knowledge into categories and sub-categories so that it can be easily remembered and retrieved.

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- ***Concept maps facilitate meaningful learning, they aid in organising and understanding new subject matter.***
- ***Concept maps are a powerful tool for identifying students' knowledge structures, especially also misconceptions or alternative conceptions.***

This helps the teacher to plan effective lessons by taking into account what a learner already knows. Students themselves gain awareness of their own knowledge organisation. Possible wrong connections in a student's knowledge become visible to the teacher and can be corrected by him/her.

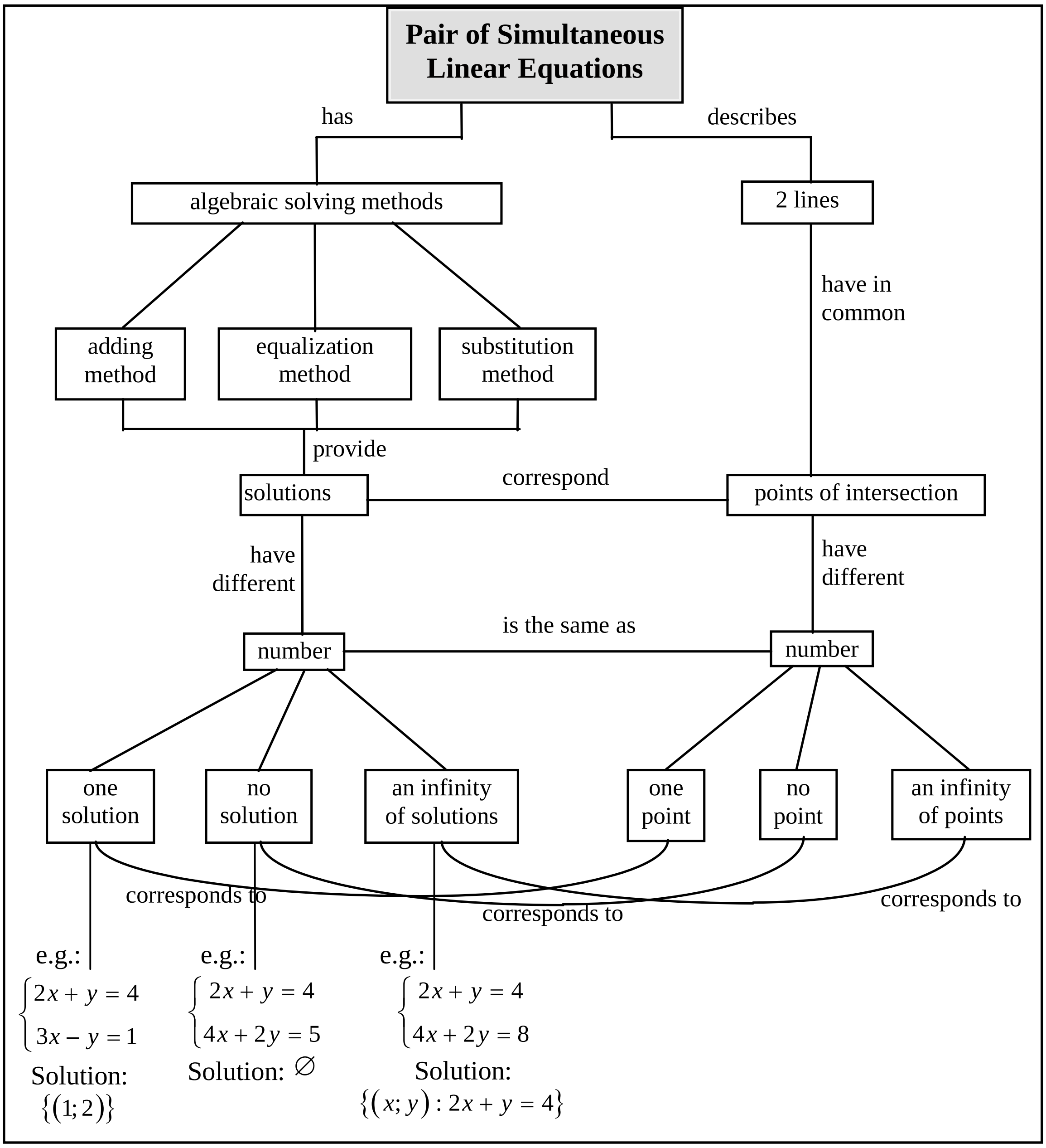

Figure 4: Concept map on the topic of linear equations

- ***Concept maps help to train the brain.***
- ***Concept maps may serve as a memory aid.***

As a concept map is a graph, a pictorial representation, it may be grasped at once, and due to its unique appearance committed well to one's memory and recalled faster.

- ***Concept maps may be used for revision of a topic.***

At the end of a topic a concept map can be constructed as repetition and in order to get a lasting and well-organised overview of a topic.

- ***Concept maps can be used for the design of instructional materials.***

Teachers found that concept maps were useful tools for organising a lecture or an entire curriculum. Moreover, they were not only aided in planning instruction, but also their own understanding of the subject matter was increased (Novak, 1996).

- ***Concept mapping may improve attitudes towards mathematics.***

By means of concept maps, an individual's mathematical knowledge may gain more structure and clarity and the individual's viewpoint on mathematics may become more positive. Furthermore, concept maps enable students through their visualisation to realise that mathematics is not a collection of isolated rules and facts but a network of ideas in which each idea is connected to several others. The authors of the Curriculum and Evaluation Standards for School Mathematics (NCTM 1989) "contend that the establishment of connections among mathematical concepts enables students to appreciate the power and beauty of the subject" (Hodgson, 1995, p. 13). Thus concept mapping may contribute to a change of an individual's beliefs on mathematics giving them a more positive emotional loading.

3.4 Limitations

It has to be considered that the method of concept mapping can be used only if one has become familiar with it. Moreover, the time that it takes to construct a concept map has to be allowed for.

In contrast to mind maps, the concepts within a concept map are linked by lines whenever they are related in some way, moreover, every single relationship is described by linking words written on the linking lines. Thus, a concept map provides much more information on a topic than a mind map, but it has not got that open structure allowing any new idea one might associate to the topic to be added easily. In addition, a concept map does not allow the same display of creativity as does a mind map.

4. Final remarks

The methods of mind mapping and concept mapping were not invented as educational tools, but it was found that these methods are useful in a variety of applications in teaching and learning processes. Yet, up to now, mind mapping and concept mapping have been rarely used in mathematics education.

However, reports about first experiences are very positive. Entrekin (1992), for example, states about mind mapping: "I found mind mapping to be an effective and delightful pedagogical tool".

The feedback of teachers that took part in further education events which I offered on the topic of mind mapping and concept mapping in mathematics is full of enthusiasm throughout. Teachers reported that students who were not good in mathematics particularly benefited from these educational tools. These students often first realised connections between mathematical concepts while producing a map. Further on, they told their teachers that only after having drawn a map they could “see” the structure of the respective mathematical knowledge. The graphical display helped the students to organise their knowledge.

Of course, depending on the pursued goals, teachers have to decide which of the two methods they particularly want to use in their lesson. The various positive learning effects that can be expected by means of both mind mapping and concept mapping ought to result in an enhanced usage of these methods in mathematics education.

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